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Difficulties Inherent in Sachs's New Theory of Elementary Matter

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Abstract

We show that the atomic energy levels, predicted by a recently proposed new theory of elementary matter, are in one-to-one correspondence with the relativistic Hartree self-consistent field theory for atomic structure. This means that this theory will contain solutions which violate the Pauli Exclusion Principle, as well as neglect the important effects of Configuration Interaction in atomic structure. This would tend to cast doubt on the viability of the new theory in its present form.

1. *Introduction and Discussion*

In several recent publications, Sachs (1961, 1963, 1965, 1969, 1971) has made constant reference to the fact that his particular formulation of a self-consistent field theory of quantum electrodynamics, based on the assumption that only 'elementary interactions' can describe physical events, contains the physical effects of the Pauli Exclusion Principle as a direct 'logical' consequence of the basic premise of the theory. In this paper, we re-examine the basic structure of the formalism developed by Sachs, and show that the solutions of the field equations actually *violate* the Pauli Principle. This will be done by showing that the energy levels predicted by this theory, for atomic structure, are in one-to-one correspondence with relativistic Hartree self-consistent field theory for atoms. Hence the Pauli Principle is absent from the energy level structure predicted in this theory. In light of this result, Sachs's argument is re-examined and is shown to contain an essential error in regard to which quantities play the role of dynamical variables in the theory.

2. *An Analysis of the Formal Structure of the Theory, and a Proof that the Pauli Principle is Violated by the Solutions of the Associated Field Equations*

In the theory under examination,† electrons and positrons are described by c-number Dirac fields coupled to each other through a classical Lorentz

† For a complete introduction to the basic premise see Sachs (1961, 1963). In particular, see the second paper listed for an extended derivation of the Pauli Principle via the 'proof by logic' technique, which we argue is invalid here.

force interaction mediated by a classical Maxwell field variable associated with each particle field $\psi^{(K)}_{(\mathbf{x},t)}$. The basic premise of the theory requires that only mutual interactions between particle fields, and not self-interactions associated with the action of a particle on itself, are the basic building blocks of the theory. In this analysis, we will simplify our argument by considering only the Lorentz force interaction and neglecting the spinor structure introduced into the Maxwell field by Sachs. This will in no way affect our conclusions since the spinor Maxwell terms have no relationship to the Pauli Principle in the theory, and may actually have been used in an inconsistent manner for other reasons.† The Lagrangian of the theory, in many ways resembling a wave-mechanical 'delayed' action-at-a-distance formalism, is

$$I = \int dx^4 \left[\sum_{K=1}^{N} \mathscr{L}_{\text{Dirac}}(\psi^{(K)}) + \sum_{K=1}^{N} \sum_{J \neq K}^{N} e^{(K)} e^{(J)} \left(\bar{\psi}^{(K)}(\mathbf{x}) \gamma^{\mu} \psi^{(K)}(\mathbf{x}) \right.\right.$$
$$\left.\left. \times \int dx'^4 S_{+}(x-x') \bar{\psi}^{(J)}(\mathbf{x}') \gamma_{\mu} \psi^{(J)}(\mathbf{x}') \right) \right] \tag{2.1}$$

The associated non-linear field equations, obtained by taking variations of (2.1) with respect to $\delta\psi^{(K)}$, are given by

$$\left(-i\partial\!\!\!/ + m + e^{(K)} \sum_{\substack{J \neq K \\ =1}}^{N} e^{(J)} \gamma^{\mu} \int dx^{4'} S_{+}(x-x') \bar{\psi}^{(J)}(\mathbf{x}') \gamma_{\mu} \psi^{(J)}(\mathbf{x}') \right) \psi^{(K)}(\mathbf{x}) = 0$$
$$(K = 1, 2, \ldots, N) \tag{2.2}$$

where $S_{+}(x-x')$ is the time-symmetric Green function of the D'Alembertian equation given by

$$\Box S_{+}(x-x') = \delta^4(x-x') \tag{2.3}$$

Equations (2.2) have eigenfunction solutions‡ of the form

$$\psi^{(K)}(\mathbf{x}, t) = \chi^{(K)}(\mathbf{x}) \exp(-iE^{(K)} t) \quad (K = 1, 2, \ldots, N) \tag{2.4}$$

† For a discussion of an apparent inconsistency in the application of the spinor Maxwell structure, as applied to the annihilation state of positronium in this model, see Frank, B. (1969). *Nuovo cimento Letters*, 1 (4), 242.

‡ Another problem with the Sachs theory lies in the stability of these stationary states; as well as that of the 'positronium annihilation state' elaborated by Sachs (1961, 1963, 1965, 1969, 1971). This is because of the presence of the time-symmetric potentials in the non-linear field equations. These are present, in analogy to formal structure of 'action-at-a-distance' electrodynamics, where the classical point trajectories are replaced by spinor wave mechanical degrees of freedom for each particle. However, in the Sachs theory, no 'complete absorber' assumption is made (a la Wheeler–Feynman) hence no retardation or radiation reaction effects occur in the theory, consequently no net transfer of energy can occur when the system is in a transient state, since time-symmetric potentials do not yield a net transfer of energy. For this reason we speculate that the stationary states of the theory, if slightly perturbed, would not be stable in time. The same goes for the vacuum annihilation state of positronium solution exhibited by Sachs. This instability of the eigenlevels would be a time-symmetric one and would not properly account for the spontaneous instability of atomic eigenlevels observed in Nature (because atoms decay downwards in energy, in a decidedly non-time symmetric fashion).

where

$$\left[\alpha . p + \beta m + e^{(K)} \sum_{\substack{J \neq K \\ =1}}^{N} \int dx'^3 \, e^{(J)} \frac{\gamma^\alpha \bar{\chi}^{(J)}(\mathbf{x}') \gamma_\alpha \chi^{(J)}(\mathbf{x}')}{4\pi |\mathbf{x} - \mathbf{x}'|}\right] \chi^{(K)}(\mathbf{x}) = E^{(K)} \chi^{(K)}(\mathbf{x}) \tag{2.5}$$

The application of Noether's Theorem† to the action principle (2.1) yields for the total energy-momentum tensor of the theory the conservation law

$$\partial^\mu T_{\mu\nu} = 0 \tag{2.6}$$

which in an eigenstate implies that the total energy ε is

$$\varepsilon = \int dx^3 \sum_{K=1}^{N} \left[E^{(K)} - \sum_{\substack{J \neq K \\ =1}}^{N} \int dx'^3 \frac{e^{(K)} e^{(J)} \bar{\chi}^{(K)}(\mathbf{x}) \gamma^\alpha \chi^{(K)}(\mathbf{x}) \bar{\chi}^{(J)}(\mathbf{x}') \gamma_\alpha \chi^{(J)}(\mathbf{x}')}{4\pi |\mathbf{x} - \mathbf{x}'|}\right] \tag{2.7}$$

The eigenfunction solutions of the theory play the role of quantum levels in this formalism, and the manifestation of the Pauli Principle occurs if these solutions obey a condition that $\chi^{(K)}(\mathbf{x}) \neq \chi^{(J)}(\mathbf{x})$ is *required* by equations (2.2), (2.5), and (2.7). However, equations (2.5) and (2.7) are related to an associated time-independent variational principle given by‡

$$\delta\left(\varepsilon - \sum_{K=1}^{N} \int dx^3 E^{(K)} |\chi^{(K)}(\mathbf{x})|^2\right) = 0 \tag{2.8}$$

which implies that the total energy is stationary, in an eigenstate, to arbitrary variations in the matter wave functions $\delta\chi^{(K)}$ subject to the constraint

$$\int dx^3 |\chi^{(K)}(\mathbf{x})|^2 = 1 \tag{2.9}$$

This is identical to the result obtained from conventional relativistic quantum mechanics, by requiring that the expectation value of the energy be stationary to arbitrary variations in the relativistic Hartree wave function

$$\Psi(\mathbf{x}^{(1)} \ldots \mathbf{x}^N) = \prod_{l=1}^{N} \chi^{(l)}(\mathbf{x}^{(l)}) \tag{2.10}$$

In this sense, equations (2.5) and (2.7) are to be recognized as the relativistic Hartree self-consistent field equations for N interacting charged Dirac particles. In the special case of a single positronium interaction (we have $N = 2$ in (2.2) through (2.7)) self-consistent Hartree solutions will exist since, in this case, $e^{(1)} e^{(2)}$ is negative. However, as is well known, the Hartree solutions cannot account for the Pauli Principle, nor can they account for

† See Bogoliubov, N. N. and Shirkov, D. V. (1959). *Introduction to the Theory of Quantized Fields*, Section 2 (New York), for a discussion of conservation laws and invariance principles related to action principles.

‡ In equation (2.8), it is assumed that (2.5) is substituted into (2.7) so that the functional dependence of $\chi^K(\mathbf{x})$ is explicit before the variation is taken.

'correlation interaction'† inherent in atomic structure. In particular solutions with $\chi^{(1)} = \chi^{(2)}$ *are* possible, and *are actually energetically preferred*, in the case of the positronium atomic levels. Hence we see that the Pauli Principle is absent from this theory, and no amount of 'logic' can force it to appear without major changes in the formalism. This difficulty is unchanged, even if the theory is applied to many-electron atoms, with infinitely massive nuclei taken as a first approximation. In this case, the structure of the eigenstates is identical to that of the relativistic Hartree theory for many-electron atoms. As is well known, the Hartree theory is unable to account for either the Pauli Principle or the 'correlation interactions' in atoms. Since the energy levels of the theory are in one-to-one correspondence with the relativistic Hartree theory, this means that the Pauli Principle is absent from the Sachs formulation and no appeal to 'logic' can cause it to appear without major modifications in the formal structure of the model.

If this is so, then how was it possible for the Pauli Principle to be derived from this theory in previous work? (See footnote †, page 199.) The answer is because a 'fundamental field' $\Psi^{(KJ)} \equiv \psi^{(K)} - \psi^{(J)}$, antisymmetric in the particle field labels, was postulated by Sachs in an *ad hoc* manner, as required in order to describe the physics of the elementary interactions in space-time. From the properties of this 'fundamental field' $\Psi^{(KJ)}$ and arguments containing non-relativistic, quasilinear assumptions, he was able to 'derive' the Pauli Principle by so-called 'logical' arguments appealing to the basic paradigm of the theory. However, the fundamental field $\Psi^{(KJ)}$ is *not* a dynamical variable in the action principle for the theory. For this reason it *never* appears explicitly in the conserved energy-momentum tensor of the theory. On the other hand $\psi^{(K)}$ *is* a dynamical variable and *does* appear in the energy-momentum tensor. For this reason *it is* $\psi^{(K)}$ and *not* $\Psi^{(KJ)}$ which controls the energy characteristics of the solutions to the associated non-linear field equations. Since his proof was based on a quantity which is not a dynamical variable of the theory, it is not surprising that results unrelated to the *actual* energy properties of the solutions to the associated non-linear field equations were derived.

The author does not basically disagree with Sachs in relation to his philosophical point of view in regard to physics. It may well be true that Nature may be describable by a non-linear field theory, whose basic elements are elementary interactions between 'observer fields' and 'observed fields'. The author objects to the fact that the present theory is misrepresented as containing *all* of the predictive potential of quantum mechanics. This is certainly not true, at least as regards the Pauli Principle. If Sachs were to recast his theory into a new form where the fundamental

† This property occurs in conventional quantum mechanics because of the presence of the coulombic potential, involving differences of particle coordinates, in the many-electron Hamiltonian. The Hartree approximation, in addition to lacking a Pauli Principle, also neglects the correlation effect because it assumes a product wave function form for the total wave function of the atom. (See J. C. Slater).

field $\psi^{(KJ)}$ appeared in the action principle as a dynamical variable, then perhaps something like a Pauli Principle might occur in the solutions to the theory. But it is not at all obvious, and the research on this remains to be done. Also, the long chain of results derived from this present formulation,† may themselves be incorrect if they rely on the assumed Pauli Principle property of the theory.

References

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† Additional derivations which may be affected by the absence of a Pauli Principle in the theory, are the existence of the annihilation state of positronium, the derivation of the black-body radiation formula from the properties of this positronium vacuum, and the derivation of the non-relativistic, antisymmetrized Schrödinger wave function for a many-electron atom, as a linearized limiting solution to the non-linear self-consistent field equations of this theory. For inconsistencies related to the calculation of the Lamb shift, within the framework of the Sachs theory, see Mann, R. A. (1968). *Nuovo cimento*, **57-B** (1), 77.